

Water, Inertial Damping, and the Complex Shear Modulus

Paul Michaels, M. ASCE, P.E.

Professor, engineering geophysics, Boise State University, Boise, ID 83725-1535;
pm@cgiss.boisestate.edu

ABSTRACT:

The author proposes an alternative to the traditional representation of soil damping. Rather than using damping ratio, this author advocates using viscosity as the specific soil property, especially for saturated permeable soils. Thus represented, the imaginary part of the complex shear modulus will vary directly with frequency. The point is particularly relevant in cases where the water table may change, thus affecting the dynamic design of foundations or structures composed of soils.

INTRODUCTION

Beginning around 1965, “equivalent viscosity” was conceived. Because resonant column studies performed on dry soil samples showed little or no viscous behavior, the sample viscosity was assumed to vary inversely with frequency (Hardin, 1965). This was done in order to continue using the Kelvin-Voigt (KV), viscous representation of the soil (which has become required under ASTM D4015). However, “equivalent viscosity” corresponds to a non-viscous viscosity. Approximately 20 years later, resonant column studies on both water saturated and dry soils revealed that the presence of water introduced a truly viscous behavior (Stoll, 1985). More recently, a Kelvin-Voigt-Maxwell-Biot (KVMB) representation of soils was published which is capable of representing damping in terms of soil permeability, porosity, and the presence of water (Michaels, 2006b). A map between the KV and KVMB representations was presented to permit the continued use of the KV formulation while taking into account the inertial coupling between soil solids and pore fluids. Such coupling is fundamental to the Biot (1956) view of wave propagation. Both the KV or KVMB models are truly viscous (friction increases with frequency).

Two Points of View, Waves and Vibrations

We begin by considering the equation for a plane, shear, S-wave in a Kelvin-Voigt (KV) continuum. The derivation appears in many text books. Let ρ be mass density, G be shear modulus, and η be viscosity. The inelastic S-wave equation is given as (Kramer, 1996)

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \eta \frac{\partial^3 u}{\partial z^2 \partial t} \quad , \quad (1)$$

where u is particle displacement orthogonal to z , the spatial axis in the direction of wave propagation. Time is represented by t . One can divide through by mass density and this produces two constant coefficients on the right hand side,

$$C_1 = \frac{G}{\rho} \quad , \quad C_2 = \frac{\eta}{\rho} \quad . \quad (2)$$

The constant, C_1 (m^2/s^2) is the square of the elastic S-wave velocity, and C_2 (m^2/s) is the viscous damping of the wave. When C_2 is non-zero, C_1 is the square of the wave's phase velocity at the limit of zero frequency.

If we move to a vibrator point of view, Kramer (1996) gives a derivation for the damping ratio, ξ , as

$$\xi = \frac{\eta \omega}{2G} = \frac{C_2 \omega}{2C_1} \quad . \quad (3)$$

The shear modulus, G , may also be expressed in terms of complex numbers. Using Kramer's notation, the complex shear modulus is given as

$$G^* = G + i\omega\eta \quad . \quad (4)$$

The loss tangent is the ratio of the imaginary to real component in Eq. 4. For a truly viscous KV medium, η is a constant, and both the complex modulus and the loss tangent will vary with frequency ω . In the case of a vibrator, the oscillator frequency will be inversely proportional to the length of a given soil volume.

The concept of "equivalent viscosity" results from assuming that the viscosity, η , is no longer constant. Rather, damping ratio is assumed constant. This new viscosity is,

$$\eta = \frac{2G}{\omega} \xi \quad . \quad (5)$$

Essentially, this is a non-viscous viscosity, a mathematical convenience which introduces other issues. It is those issues which this paper is about. The convenience is that the complex shear modulus can now be written in terms of a constant damping ratio,

$$G^* = G (1 + i2\xi) \quad . \quad (6)$$

Engineering practice has been to treat damping ratio as a specific material constant (like density or the real part of the shear modulus).

Historical Background

The continuum representation can also be recast in terms of lumped parameters. This permits springs and dashpots to be employed in the analysis. Fundamentally, soils are represented by both stiffness (spring) and viscous damping (dashpot). Viscous behavior is evident when the resistance to motion varies with the particle velocity. Since particle velocity will increase with oscillatory frequency, we expect frequency dependence for key measures of motion, like damping ratio, loss tangent, phase velocity, and amplitude decay of a wave.

Fig. 1 (a) and (b) shows the KV representation for both the vibrator and wave points of view. In the case of a vibrating soil mass, we expect a sinusoidal exponential decay with time of the

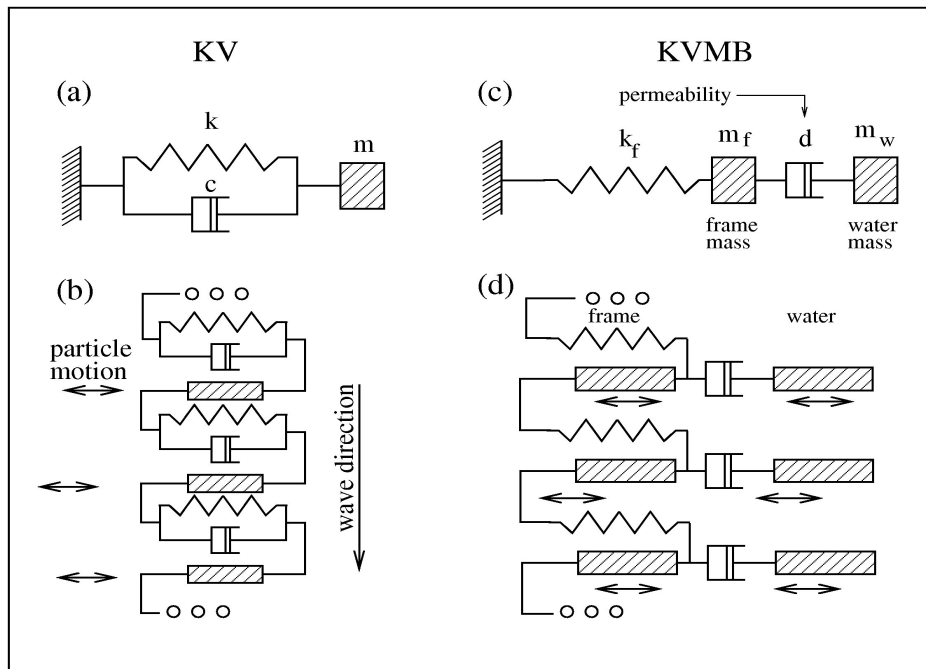


Fig. 1. KV (a) vibrator, (b) wave assemblage and KVMB (c) vibrator, (d) wave assemblage.

motion due to an impulsive excitation. The center frequency of vibration will depend on the length of the soil volume.

Assemblages of the single degree of freedom element may be used to represent the wave point of view. In the wave view, we expect the wave to exponentially decay in amplitude as a function of distance propagated. Further, we expect that the phase velocity and rate of decay to be both functions of frequency.

However, early attempts to measure the viscous behavior of soils showed little or no measurable frequency dependence of the motion. Hardin (1965) found that dry soils evaluated with what has become known as the resonant column test, ASTM-D4015 (ASTM, 1996) did not present the expected viscous behavior. Engineers conducting soil testing found themselves caught between a desirable mathematical representation and the conflicting reality of the measurements. Hardin proposed a solution in the synopsis of his 1965 paper.

“. . . the viscosity should be assumed to decrease with frequency such that the ratio, viscosity times frequency divided by shear modulus, is a constant with frequency, in order to use this model. Values of the ratio for dry sands are given.”

This solution is essentially Eq. 5 above. The model Hardin referred to was the KV representation, and the measured sands were dry. The soil cylinder was strained in rotational shear. This became the standard way of doing the test, and the conclusion about viscosity lead to the definition of “equivalent viscosity”. The concept of equivalent viscosity also worked its way into computer software like SHAKE (Schnabel et al., 1972).

Applying resonant-column tests on water saturated samples would occur later. Stoll (1985) compared the shear behavior of resonant-column tests on both dry and saturated soils. While the loss tangent was constant with frequency for dry soils, the same soils, when water saturated, demonstrated measurable variation of loss tangent with frequency. Thus, damping ratio could no longer be held as a constant.

This author has observed these same, behaviors between dry and saturated soils in down-hole shear wave surveys (Michaels, 1998; Michaels, 2006a). The long held contention that pore fluids may interact with the soil frame producing a viscous damping makes particular sense when the medium is permeable. Further, the amount of friction due to relative fluid-frame motion might be expected to be greater with increasing pore size, up to some limit where the friction might begin to drop off in very large pore spaces. Stoll's (1985) measurements demonstrated greater viscous behavior for the sand than was observed for the silt.

KVMB, An Alternative Representation

An alternative representation to the Kelvin-Voigt (KV) model was published for the specific case of shear strain (Michaels, 2006b). The representation was termed Kelvin-Voigt-Maxwell-Biot (KVMB) since it was based on the ideas of those authors. The KVMB representation splits the combined mass into two components (pore fluid and solid frame). These two masses are connected by a dashpot which can then be related to permeability. The KVMB model can also be used to represent both vibrations and wave propagation, Fig. 1 (c) and (d). A mapping between the KV and KVMB representations is included in Michaels (2006b). With this mapping, one may continue to use the KV model, and the mapping provides a connection to soil properties like permeability and porosity.

DOMAINS OF APPLICATION

Pore Fluid Density and Inertial Coupling

One might expect that, since air is also a viscous fluid, viscous behavior should also be observed for dry soils. Indeed, this seems to be the case when the soil is subjected to compressive strain (Michaels, 2006a). However, in the case of shear, the mechanism is inertial, and the low density of air significantly reduces the resistance to motion that an air mass can produce. An analogy would be this. Consider squeezing a sponge to remove either the air or water in the pores. Then consider shaking the sponge. Squeezing (compression) is far more effective in producing a relative motion between the fluid and the solid. The image of a dog shaking its wet fur dry may also be helpful.

Using a truly viscous representation is important in shear when the pore fluid is dense, and permeability is in a range that will permit relative motion between the fluid and frame under an inertial drive. Thus, water saturated and permeable soils require a truly viscous representation, while dry soils apparently do not.

Damping Ratio vs. Viscosity as a Soil Property

Engineering practice tends to follow the equivalent viscosity model and represent the soil damping property in terms of a constant damping ratio (Rix et al., 2000). Often, numerous studies are cited for this decision. For those numerous studies which were conducted on dry samples, or samples of low permeability, this is easy to understand. Most resonant column work is done on dry (air saturated) samples, so it is not surprising that damping ratio has

been considered a constant. The thesis of this paper is two fold:

1. Water saturated soils are not well represented by a constant damping ratio, but by constant viscosity if they are permeable. Water saturated soils are different from dry soils, and engineering design needs to consider the water table.
2. One should refrain from using the damping measurements taken from tests on dry soils when the field conditions are expected to be water saturated and permeable. If tests are done on saturated samples, one may either overestimate or underestimate the damping. Under the constant damping ratio assumption, these possibilities exist due to the frequency dependence of damping ratio under saturated conditions.

In this author's view, using a truly viscous representation will always be best, even when not necessary. Dry soil viscosities can always be represented by very small viscosities. The "equivalent viscosity" representation does not enjoy the same breadth of applicability, and can not represent the frequency effects presented by permeable, water saturated soils.

FIELD DATA EXPERIENCE

As mentioned above, a mapping was derived between the KV and KVMB representations (Michaels, 2006b). This mathematical map is achieved by eigenvalue decomposition of the governing differential equations for the two vibrator representations. Fig. 2 illustrates how a traditional KV damping ratio can be related to hydraulic conductivity under the KVMB model for the down-hole field data published in Michaels (1998). There are a number of steps.

1. Invert the wave measurements of velocity dispersion and amplitude decay to obtain C_1 and C_2 (Michaels, 1998).
2. Use Eq. 3. to compute a KV damping ratio for a vibrator of some defined thickness of soil (and hence frequency). The choice of length can be set to match the center frequency of the seismic wavelet. It is not a critical decision as long as one is on the coupled side of the mapping (Fig. 2).
3. For a given porosity and frequency, find the point on the KVMB mapping curve with an ordinate equal to the KV damping ratio of step (2).

The field data from Michaels (1998) were applied to the above procedure, and cases for a silt, sand, and gravel are shown in Fig. 2 (and Table 1). Several possible porosity curves are shown, since

Mapping of published down-hole data (Michaels, 1998)

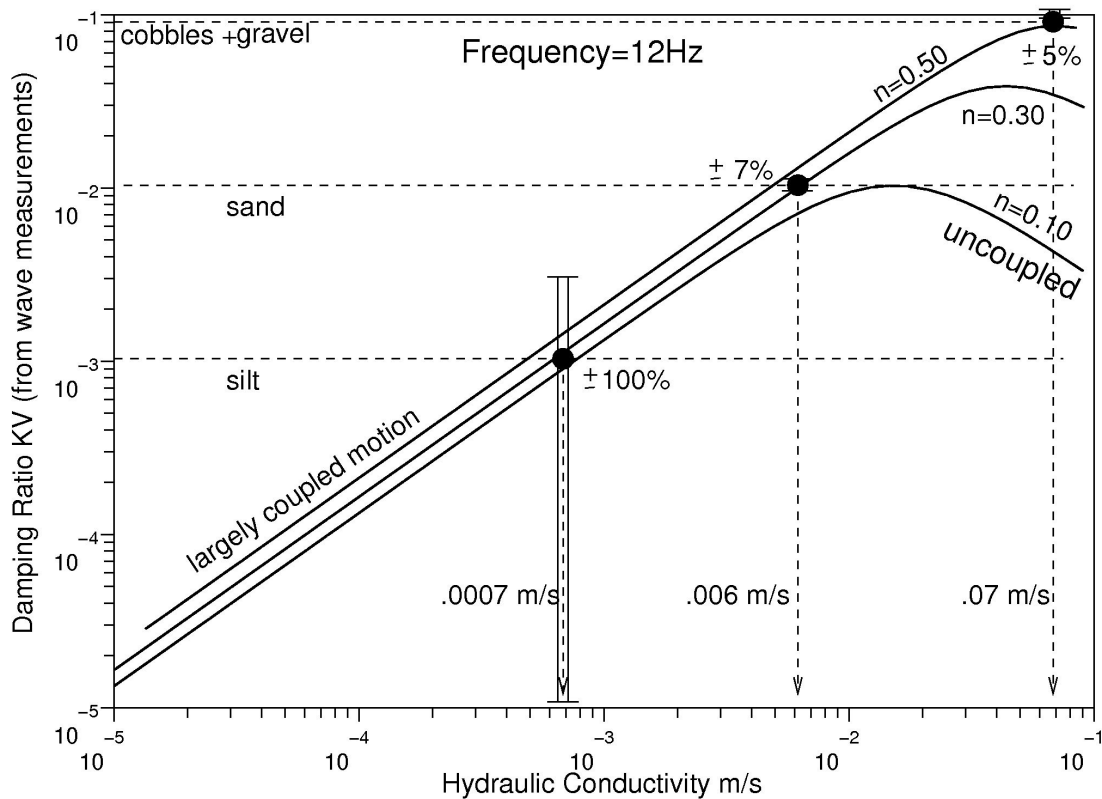


Fig. 2. Plot of 3 soil samples on the computed map between KV damping ratio and hydraulic conductivity. Mapping is for 3 possible porosity cases (10%, 30%, 50%). Confidence limits are 95% for KV damping ratio computations.

Table 1. KV Damping Ratio from C₁ and C₂ at 12 Hz (Michaels, 1998)

Soil	C1 (95%conf.)	C2 (95%conf.)	KV D. Ratio	D.R. 95% Conf.
Logan Silt	25567 +/-218	1 +/- 1	.00147	+/-100%
Logan Sand	51343 +/-375	14 +/- 1	.01028	+/- 7.2%
Idaho Gravel	94917 +/-2913	255 +/- 9	.10128	+/- 4.6%

measurements of soil porosity were not available. In the case of the Geologan 1997 data, soil type was determined from cone penetrometer soil behavior type (SBT). Soil sampling was done in the case of the Idaho data. At 12 Hz, silts are at the limit of detection for damping (large error bars). Also, the degree of water saturation may have been less than 100% at the Logan site. Thus, silts could easily be considered non-viscous at these and lower

frequencies. Sands and gravel, on the other hand, produce measurable damping effects. The 95% error bars on the computed damping ratio are 7.2% and 4.6% of the measured value for these soils respectively.

Domain of Equivalent Viscosity

From Fig. 2 we see that, at 12 Hz, the mathematics predicts largely coupled motion between the soil water and solid frame regardless of grain size. Fine grained soils, like silt, will be less permeable than the granular soils, and we expect far less friction to develop, making soils like silt comparable to dry soil in terms of dynamic behavior. This map is for a 12Hz vibrator. For a silt, significantly higher frequencies would be needed to shake the water into motion relative to the frame. Given that most earthquake motions are at lower frequencies, it is not likely that much harm would be done with an equivalent viscosity analysis in the case of a silt. Vibrations from blasting might be a different matter.

Domain of True Viscosity

We can see from Fig. 2 that as grain size increases, the degree of damping also increases (larger KV damping ratio). The advantage of specifying the soil profile in terms of viscosity instead of damping ratio is that dynamic calculations will automatically adjust for what ever frequencies are expected during an earthquake. The importance of this varies with pore fluid. Using viscosity instead of a single damping ratio benefits one most when water is the fluid, and permeability is large. In any case, a truly viscous representation will work for all soils. When the soil is of low permeability, the viscosity will just be a very small number.

Another observation might be that laboratory tests on water saturated samples of low permeability might require forced excitation at high frequencies to obtain a good measurement. At low frequencies, the damping ratio will be difficult to measure, and quite small in value. Fig. 3 illustrates how shaking the water saturated silt at about 100 Hz should produce the same degree of frictional loss as the sand shaken at 12 Hz.

CONCLUSIONS

The current use of “equivalent viscosity” will certainly serve the engineering communities needs when the pore fluids are of low density (ie. air), or if the soil is so impermeable that viscous effects are prevented by lack of relative motion between the pore fluid and

Predicted KV Damping Ratio as a Function of Frequency

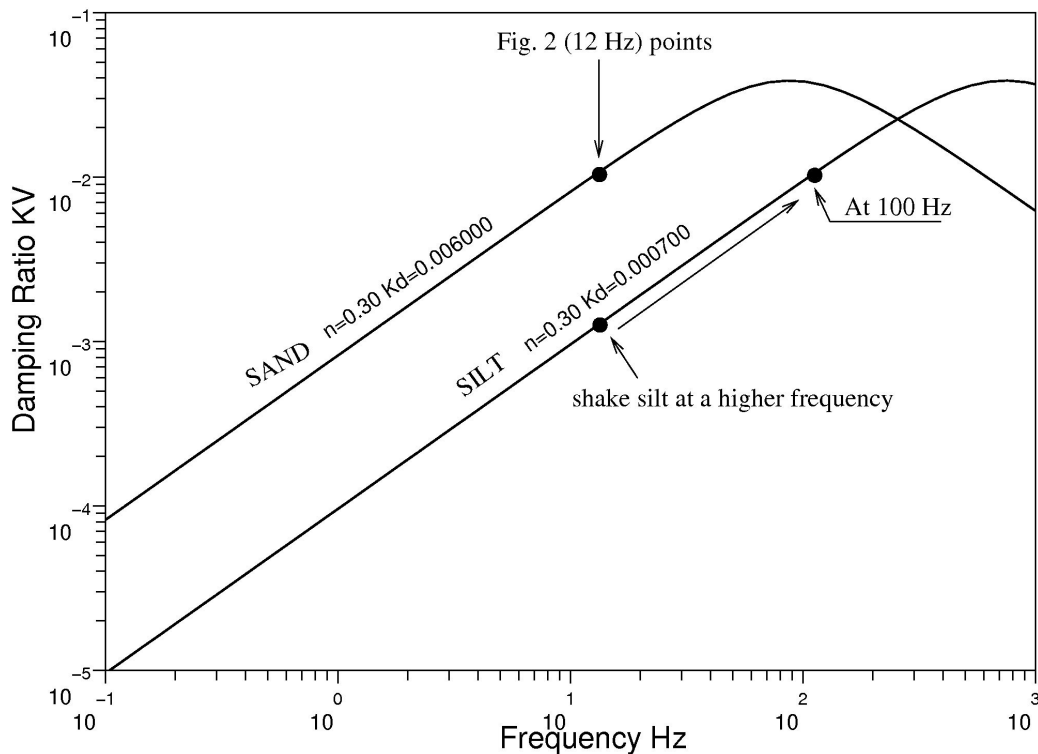


Fig. 3. Improved confidence in the measurement of a silt's damping is predicted by the KVMB model if the frequency is increased.

frame. However, in permeable soils, the equivalent viscosity approach will not be able to capture the variation in damping ratio as a function of frequency. The worse case would be to measure a damping ratio on a dry sample of a granular soil, and then apply that result to a water saturated design problem.

Using a “truly viscous” representation will be appropriate over all domains. Dry or impermeable soils would just present very small values of viscosity, η , or damping coefficient C_2 . However, best results would occur by determinations of soil stiffness and damping under the conditions of the field design problem. This requires knowledge of the water table and its variation with time.

Improvements in the measurement of damping can be had by selecting a vibration frequency that results in a large value of damping. At low frequencies, the less permeable soils may present damping values below reliable measurement capabilities.

ACKNOWLEDGMENTS

The author would like to thank the ASCE GeoInstitute and Utah State University for providing the field site and borehole at the Geologan 1997 conference. In particular, Dr. Caliendo of the Utah State Civil Engineering Department did a great job supervising the drilling and management of the field day. Thanks also go to S-Sixteen Inc., Anderson Associates Inc. and Dr. Warren Barrash for providing access to the Idaho borehole.

Computer data processing was done using open source software running in the Linux operating system. There are too many contributors to give individual credit.

REFERENCES

- ASTM. (1996). "Standard test methods for modulus and damping of soils by the resonant-column method." *D4015*, West Conshohocken, Pa.
- Biot, M. (1956). "Theory of propagation of elastic waves in a fluid saturated porous solid. I: Low-frequency range" *J. Acoust. Soc. Am.*, 28(2), 168-178.
- Hardin, B.O. (1965). "The nature of damping in sands." *J. Soil Mech. Found. Div.*, Vol. 91 (1), 63-97.
- Kramer, S.L. (1996), *Geotechnical earthquake engineering*, 1st Ed., Prentice-Hall, Upper Saddle River, N.J.
- Michaels, P. (1998). "In situ determination of soil stiffness and damping." *J. Geotech. Geoenviron. Eng.*, Vol. 124 (8), 709-719.
- Michaels, P. (2006a). "Comparison of Viscous Damping in Unsaturated Soils, Compression and Shear." *Unsaturated Soils 2006, Vol. 1, Proceedings of the Fourth International Conference on Unsaturated Soils*, (GSP 147), ASCE, Reston, VA. 565-576.
- Michaels, P. (2006b). "Relating damping to soil permeability" *Int. J. of Geomechanics*, Vol. 6 (3), 158-165.
- Rix, G.J., Lai, C.G., and Spang Jr., A.W. (2000). "In situ measurement of damping ratio using surface waves." *J. Geotech. Geoenviron. Eng.* Vol. 126 (5), 472-480.
- Schnabel, P.B., Lysmer, J., and Seed, H.B. (1972). "Shake, A computer program for earthquake response analysis of horizontally layered sites." *Report No. EERC 72-12*, U. of California, Berkeley
- Stoll, R. (1985). "Computer-aided studies of complex soil moduli." *Proc., Measurement and Use of Shear Wave Velocity for Evaluating Dynamic Soil Properties*, ASCE, New York, 18-33.